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MATHEMATICAL MODEL FOR DETERMINING THE HYDRAULIC CHARACTERISTICS OF FINELY DISPERSED WATER MINERAL SUSPENSIONS

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Abstract. Hydraulic characteristics of polydisperse mineral suspensions such as viscosity, concentration, porosity, volume and weight content of solid and liquid phases are necessary to calculate the speed of free or constrained deposition and floating of particles of different composition and size. This speed is the basis for the calculation of hydraulic classifiers and separators for the enrichment of mineral pulps. Determination of hydraulic characteristics requires a lot of experimental measurements, taking into account the different composition of suspensions and operating modes of the devices. The known calculation formulas are empirical and semi-empirical. Theoretical formulas are known only for viscosity, but they are limited by the concentration of the solid phase within 2–5%. The aim of the work is to develop a mathematical model for determining hydraulic characteristics depending on only one measured indicator – the density of the suspension (the volume weight of the sample). This indicator is easily measured in practice, at processing plants it serves to monitor the operating mode of the devices. In this work we use a cellular model of a water suspension consisted of discrete particles, and classical definitions of hydraulic characteristics. Based on this, defining formulas were obtained, an algorithm and a program for calculating characteristics were developed. When using the program, the obtained database allows us to establish approximating dependences: for the weight content of the solid phase θ, porosity ε, concentration β, kinematic viscosity v, density of the suspension ρ_s in a wide range. These dependencies allow us to calculate the hydraulic characteristics for any zone of the apparatus and different modes using only one simple measurement of pulp density by the weight method. Based on this, for example, it is possible to calculate the speed of constrained deposition and floating of particles and to build a map of the distribution of speeds and the efficiency of gravitational separation of particles. The developed mathematical model, algorithm and calculation program can be used to evaluate the optimal mode, control the stability of the equipment and design new hydraulic devices.

Keywords: mineral suspension, density, viscosity.

1. Introduction

To calculate the design and operating modes of hydraulic devices for classification and separation of mineral suspensions, the determining factor is the speed of constrained particle movement. In the simplest case, for thickeners, deslimers, settling tanks, this is the speed of gravitational constrained deposition and floating. The efficiency of separation of fine suspensions under gravity is determined by viscosity, size, shape, density, concentration of solid particles, etc. It is impossible to separate suspensions having the same densities of the liquid phase and solid particles, as well as with a particle size of 5-10 µm for which Brownian motion prevails over gravitational deposition. The efficiency of hydraulic separation decreases with an increase in the concentration of particles and the viscosity of the suspension.

When describing processes in hydraulic apparatuses, a cellular model is mainly used. There suspension consists of solid particles with intervals between them. It is assumed that the particles interact weakly with each other, the velocity gradient does not depend on the viscosity, and at a significant concentration the particles have a certain packing (cubic, rhombohedral etc.). The cellular model in the form of an ensemble of balls, in comparison with the capillary model, allows us to consider sparse granular layers [1, 2].

Within the framework of the cellular model, many formulas have been proposed for the speed of constrained particle movement [1–3]. The unifying factor is that in all known formulas such hydraulic characteristics as kinematic (or dynamic) viscosity ν and porosity ε are used. Porosity is the fraction of the gaps between the particles that are filled with liquid, or the fraction of the liquid phase. An equivalent concept is often used - the concentration or fraction of the solid phase $\beta = 1-\varepsilon$.

Experimental determination of the characteristics of ε and ν requires a variety of measurements, taking into account the variety of suspensions type and modes of operation of the devices.

For viscosity ν, the well-known theoretical formulas of A. Einstein, L.D. Landau are valid at a low volume concentration of solid phase $\beta = 2 - 5\%$. [4]. Basically, all expressions for ν are empirical and semi-empirical and are limited in concentration [2]. For example, the semi-empirical formulas of Vakhrushev and Zuber are limited to a solid concentration of up to 25% [3].

To calculate the porosity ε, in the 50s and 60s, Epstein reviewed more than 50 formulas, and Rutgers analyzed a large number of ratios between concentration and viscosity [5]. However, the problem of correctly calculating the characteristics of ν and ε with the involvement of a minimum of experiments remains relevant and it still attracts the attention of scientists [6–8].

Practice of measuring hydraulic characteristics. During enrichment, the pulp is a relatively dense unstructured mineral suspension in relation to the particles moved in it. In the practice of mineral processing, two hydraulic characteristics of mineral pulps are constantly measured to control technological processes: the weight content of the solid phase or the percentage of solid θ and the density of the pulp ρ_s. The percentage of solid θ is the weight of the solid fraction relative to the weight of the wet sample. To determine θ, a wet sample is weighed, then dried in a drying cabinet, the dry residue is weighed, the calculation θ is performed according to the formula:

$$\theta = \frac{P_m}{P_m + P_{liq}} \cdot 100 \qquad (1)$$

where P_m, P_{liq} – the weight of solid and liquid in the pulp, respectively.

Also, in the practice of enrichment, the density of various pulps is constantly monitored: sands and drains of mills, classifiers, separation products, desliming, filtration etc. The suspension density ρ_s or volume weight (g/cm^3) is defined as:

$$\rho_s = \frac{P_m + P_{liq}}{V_m + V_{liq}} \qquad (2)$$

where V_m , V_{liq} – the volume of solid and liquid in the pulp, respectively.

In practice, the density ρ_s of the pulp is easily measured by weighing a sample of the suspension, which was collected in a measuring cup with a capacity of 1 liter. Such a sample can be taken at any point of the hydraulic apparatus, the ρ_s is usually from 1.1–1.2 g/cm^3 to 1.6–1.7 g/cm^3.

Main idea. The idea of the work is to establish the dependence of the hydraulic characteristics of only one indicator – the volumetric density of the suspension. This idea is based on the statement of J.F. Richards [2] that "the true gravity is equal to the weight of solid particles minus the pushing force, and the pushing force depends more on the density of the suspension, than on the density of the liquid, that is part of the suspension, since the vertical gradient of the statistical pressure (drag force) in the suspension is determined precisely by its density". It follows from this, that the speed of movement (determined from the balance of forces) and auxiliary values (viscosity, etc.) are advisable to associate with the density of the suspension, but not with density of the liquefying agent, which in most hydraulic devices is water.

The idea of expressing all hydraulic characteristics in terms of ρ_s is attractive because measuring "manually" the suspension density of ρ_s for any pulp, then it can be easy to calculate the remaining hydraulic characteristics - percentage of solid, porosity, viscosity, and then the speed of deposition and floating. For example, taking samples at different points of the working area of the device, you can build a real picture of the distribution of speeds.

The aim of the work is to establish analytical dependences of hydraulic characteristics (solid content, porosity, viscosity etc) on one indicator that is easily measured in practice - on the density of the suspension ρ_s. These dependencies will allow us to calculate the speed of particle motion, which is necessary to control the operating mode of hydraulic devices and design the design of new devices.

2. Methods - theoretical analysis and mathematical calculations.

3. Results and discussion

Mathematical model.

Analytical expressions of the mathematical model were obtained on the basis of classical definitions of hydraulic characteristics ρ_s , θ, ε, β, ν.

In equation (2), we express ρ_s in terms of θ from equation (1). Note that $V_{liq}= T_{liq}/\rho_{liq}$, $V_m= T_m/\rho_m$, where ρ_m, ρ_{liq} – the densities of the solid and liquid phases of the suspension, respectively. At the same time $\rho_{\text{liq}}= 1$ g/cm^3 since when enriching mineral suspensions, we almost always deal with water solutions. After the transformations, we get:

$$\rho_s = \frac{1}{1-\frac{\theta}{100}\left(1-\frac{1}{\rho_m}\right)} \qquad (3)$$

The density of the suspension in (3) is measured in g/cm^3, to convert to kg/m^3, ρ_s is multiplied by 1000.

We obtain an analytical expression for the characteristic θ. It will allow you to calculate θ according to the known density of the suspension ρ_s. From formula (3) we have:

$$\theta = 100 \cdot \frac{\rho_m \cdot (\rho_s - 1)}{\rho_s \cdot (\rho_m - 1)} \quad (4)$$

Hydraulic characteristic porosity ε (units) determines the relative volume of liquid in the gaps between the particles. This is the volume fraction of the liquid, or the volume concentration coefficient of the liquid in the suspension:

$$\varepsilon = \frac{V_{liq}}{V_m + V_{liq}} \quad (5)$$

The volume fraction of a solid or the volume concentration coefficient of a solid β (units) is defined as:

$$\beta = \frac{V_m}{V_m + V_{liq}} \quad (6)$$

Obviously, $\varepsilon + \beta = 1$ and $\varepsilon = 1 - \beta$.

We obtain an analytical expression for ε as a function of ρ_s.

Let's write the formula (5) using expressions (1) and (4):

$$\varepsilon = 1 - \frac{\rho_s}{\rho_m} \cdot \frac{\theta}{100} \quad (7)$$

From formulas (7), (4) we have:

$$\rho_s = \rho_m \cdot (1 - \varepsilon) + \varepsilon \quad (8)$$

Finally, ε is defined from (8) as:

$$\varepsilon = \frac{\rho_m - \rho_c}{\rho_m - 1} \quad (9)$$

As the concentration in the suspension increases, the interaction of particles increases, but as long as they do not form a constant structure, the suspension can be considered a Newtonian liquid. The state of the pulp during enrichment does not show the presence of a structure in which the particles are oriented relative to each other in a certain way. That is, the pulp, as a "liquid-solid" system, is a polydisperse, non-structured Newtonian liquid and the viscosity formulas of Newtonian liquids are suitable for it.

Among the many well-known formulas for *kinematic* viscosity, it is advisable to choose the formula of Vend V. (1948) [4]. It covers the widest range of concentrations β, and it was obtained based on the study of various pulps and is in good agreement with experimental data for all concentrations at which the fluidity of the suspension is still preserved:

$$\nu = \nu_0 \cdot exp \frac{2.5 \cdot \beta + 0.675 \cdot \beta^2}{1 - 0.609 \cdot \beta} \quad (10)$$

where ν - cm^2/s, ν_0 =0.01 cm^2/s is the kinematic viscosity of water at 20 °C.

The analytical expression for viscosity as a function of the density of the suspension ρ_s is obtained as follows.

First you need to build a dependency in conditional coordinates (x, y) = [ρ_s, (100-1/ν)], where ν is determined by the formula (10). For this dependence, we determine the approximating (correlation) function F(x, y) and then calculate the dependence $\nu = f(\rho_s)$ in the form ν = 1 / (F +100). The rationale for this approach is obtained in [7].

Note that to account for the size and shape of the particles, correction coefficients are introduced to the hydraulic characteristics θ, ε, ν. Most often, relative geometric characteristics are used as coefficients - the cross-sectional area of the particles, the equivalent diameter, etc. For example, in [8], when determining ε, the material is divided into fractions – large, medium, small, and for each fraction for ε, their own correction coefficients are introduced.

The hydraulic devices receive raw materials after grinding and classification, so it has a relatively small variation in size. The effect of particle size on the hydraulic characteristics of the suspension is a separate task. Whereas the size of a single particle moving in a suspension is explicitly included in all formulas for calculating the speed of constrained deposition. Because of this, in the first approximation, the influence of particle size and shape on hydraulic characteristics can be ignored.

***Particle density of the solid phase of the suspension*.**

All the above formulas (1) – (10) explicitly or implicitly include the indicator ρ_m – the density of solid phase particles. For a monodisperse suspension, it is obvious that this is the density of the particle material. For a polydisperse suspension, ρ_m can be determined by two characteristics measured in practice – ρ_s and θ. From formula (3) we get:

$$\rho_m = \frac{1}{1 + \frac{100 \cdot (1 - \rho_s)}{\theta \cdot \rho_s}} \quad (11)$$

Indicators ρ_s and θ of various pulps are constantly monitored at processing plants to control the technological regime. The task is to keep these indicators constant, optimal for this operation and the device. Thus, according to the fluctuations of ρ_s and θ, and, accordingly, ρ_m, the change in the hydraulic properties of the pulp will be monitored, that is, the fluctuation of ρ_m relative to some average value. For the purposes of calculating and designing devices, it is convenient to define this average as a weighted average.

For multicomponent pulps, we recommend defining ρ_m as the weighted average density of the constituent components:

$$\rho_m = \frac{\sum_{i=1}^{n}(\rho_i \cdot \gamma_i)}{\sum_{i=1}^{n}\gamma_i} \qquad (12)$$

where $i = 1, \ldots, n$ – the number of particles of different grades in the suspension; ρ_i and γ_i – density and output of particles i–grade, respectively.

Obviously, the ρ_m calculated by the formula (12) can be corrected using (11) and vice versa. An example of determining the density of solid phase particles ρ_m for some polydisperse suspensions is given in Table 1.

Table 1 – Weighted average density of solid phase particles ρ_m of some finely dispersed water mineral suspensions

Iron ore processing waste			Waste from the basalt quarries of Volhynia		
Component	Output γ_i, %	Density ρ_i, g/cm^3	Mineral	Output γ_i, %	Density ρ_i, g/cm^3
Iron (chemical)	12.3	7.8	Tuff	15	1.4
Quartz	57.7	2.65	Lava-breccia	68	2.2
Others	30	2.2	Basalt	17	2.6
ρ_m of suspension		3.15	ρ_m of suspension		2.15

Model limitation.

The limitation of the characteristics from below means that there are practically no particles in the suspension, the density of the suspension ρ_s tends to 1, the density of water, and θ tends to zero. The restriction from above corresponds to the most dense packing of particles. K.F. Gauss proved that rhombohedral packing was the densest for spherical particles. At the same time, the maximum volume concentration coefficient of solid β = 0.741 units, and the minimum porosity ε = 1 - 0.741 = 0.259. The values of β and ε determine the restrictions from above. Based on formulas (7) and (8) we get:

$$\rho_s = \rho_m \cdot (1 - \varepsilon) + \varepsilon \;\rightarrow\; \rho_{s\ limit.} = \rho_m \cdot 0.741 + 0.259 \qquad (13)$$

$$\varepsilon = 1 - \frac{\rho_s}{\rho_m} \cdot \frac{\theta}{100} \rightarrow \qquad \theta_{lim} = 74.1 \cdot \frac{\rho_m}{\rho_s} \qquad (14)$$

As we can see, the limiting values of density and percentage of solid depend on the grade of particles ρ_m. It follows from the Wend formula (10) that for a suspension of solid discrete particles at β = 0.741, the maximum achievable viscosity of the granular layer does not depend on the type of particles and is ν = 0.5748 cm^2/s.

Table 2 shows an example of calculating the boundary values of ρ_s, θ for some types of suspensions

Table 2 – Upper boundary value of the characteristics of ρ_s, θ at different densities of solid phase particles of the suspension ρ_m

Suspension	ρ_m, g/cm^3	$\rho_{s\ lim}$, g/cm^3 at (13)	θ_{lim}, % at (14)
Iron ore processing waste	3.15	2.593	90
Waste from basalt quarries	2.15	1.852	86.0
	2.0	1.741	85.1

The high boundary values of ρ_s and θ, are given in Table 2, include practical indicators of the operation of hydraulic devices. Thus, in the sands of spiral classifiers, θ is up to 70–75%, in the sands of hydrocyclones - up to 80%, the cake density of vacuum filters is ρ_s =1.8–1.85 g/cm^3 [1]. In ore-dressing practice the pulp density of ~ 2.0 g/cm^3 and higher is never realized, since classification and separation processes are difficult in a thick suspensions. Thus, the cellular model used is suitable for practical calculations, it covers a wide range of ρ_s and θ during enrichment.

***Algorithm and calculation program*.**

The algorithm for calculating and determining the approximating dependencies of hydraulic characteristics on the density of the suspension is as follows.

1. We determine the weighted average density ρ_m (g/cm^3) by the formula (11);
2. We determine the boundary values: ρ_s by the formula (12) and θ by the formula (13);
3. We set the initial value of ρ_s, for example, 1.1 g / cm^3, then we vary it with an arbitrary step up to $\rho_{s\ lim.}$;
4. For each of the ρ_s, we define θ_i (%) by the formula (4);
5. We define the dependence $\theta_i = f\ (\rho_{s\ i})$. To do this, we build a trend with the maximum value of the square of the correlation coefficient R^2 and determine the approximating function of the trend;
6. For each θ_i according to p.4, we determine ε_i (units) by the formula (7);
7. We determine the dependence $\varepsilon_i = f\ (\rho_{s\ i})$. To do this, we build a trend with the highest R^2 and determine the approximating function of the trend;
8. For each ε_i, according to p.6, we calculate β_i (items) by the formula $\beta_i = 1 - \varepsilon_i$;
9. For each β_i according to p.8 we calculate ν_i by the formula (10);
10. We determine the approximating dependence $\nu_i = f\ (\rho_{s\ i})$. For this:

    10.1 Calculating a conditional variable y_i =(100–1/ν_i);

    10.2 Building a trend in conditional coordinates (x_i, y_i) =[$\rho_{s\ i}$, (100–1/ν_i)]. At the highest R^2, we determine the correlation function of the trend $F\ (x_i, y_i)$;

    10.3 Calculating the dependency $\nu_i = f\ (\rho_{s\ i})$ in the form $\nu = 1/(F+100)$.

This algorithm is implemented in the program Mc. Excel (Tables 3, 4).

Table 3 – Database of hydraulic characteristics at ρ_m =3.15 g/cm^3

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
1	**3.15**							
2	**ρ_s, g/cm^3**	***θ*, %**	***ε*, *item.***	***β*, *item.***	***ν*, cm^2/s**	**100-1/*ν***	**Q $_{solid}$, t/h**	**Q $_{liquid}$, m^3/ h**
3	1.1	13.3	0.953	0.047	0.011	11.4	50	0.1
4	1.2	24.4	0.907	0.093	0.013	22.3	50	0.1
5	1.3	33.8	0.860	0.140	0.015	32.7	50	11.8
6	1.4	41.9	0.814	0.186	0.017	42,4	50	16.2
7	1.5	48.8	0.767	0.233	0.021	51.3	50	20.7
8	1.6	54.9	0.721	0.279	0.025	59.5	50	25.5
9	1.7	60.3	0.674	0.326	0.030	66.9	50	30.6
10	1.8	65.1	0.628	0.372	0.038	73.4	50	36.0
11	1.9	69.4	0.581	0.419	0.048	79.1	50	41.7
12	2	73.3	0.535	0.465	0.062	83.9	50	47.7
13	2.2	79.9	0.442	0.558	0.114	91.2	50	61.0
14	2.4	85.5	0.349	0.651	0.239	95.8	50	76.0
15	2.5	87.9	0.302	0.698	0.367	97.3	50	84.4
16	**2.593**	**90.0**	**0.259**	**0.741**	**0.575**	**98.3**	**50**	**103**

Table 4 – To build a database Table 3

№	Cell	Formula	Expression in the program
1	A1	ρ_m , g/cm^3	A1= 3,15
2	A16	$\rho_{s\ lim} = \rho_m \cdot 0.741 + 0.259$	A16=$A$1*0,741 +0,259
3	B16	$\theta_{lim} = 74.1 \cdot \rho_m / \rho_s$	B16=74,1*$A$1/A16
4	A3… A15	$\rho_s, \ldots, \rho_{s\,i} \le \rho_{s\ lim}$	A3=1,1, A4=A3+0,1, … A15 < A16
5	B3… B15	$\theta = 100 \cdot \dfrac{\rho_m \cdot (\rho_s - 1)}{\rho_s \cdot (\rho_m - 1)}$	B3=100*((($A$1*(A3-1))/(A3*($A$1-1))))
6	C3…C16	$\varepsilon = 1 - \dfrac{\rho_s}{\rho_m} \cdot \dfrac{\theta}{100}$	C3=1-((A3*B3)/($A$1*100))
7	D3… D16	$\beta = 1 - \varepsilon$	D3=1-C3
8	E3… E16	$\nu = \nu_0 \cdot exp\dfrac{2.5 \cdot \beta + 0.675 \cdot \beta^2}{1 - 0.609 \cdot \beta}$	0,01*DEGREE (2,7183;(2,5*D3+0,675*D3^2)/(1-0,609*D3))
9	F3… F16	$100 - \dfrac{1}{\nu}$	F3=100-1/E3
10	G3… G16	Q_{solid}, t/h	G3= 50 =G4=…=G16
11	H3 …H16	$Q_{liquid} = \dfrac{Q_{solid} \cdot (100 - \theta)}{\theta}$, m^3	H3 =G3(100-B3)/B3

The value A1 = ρ_m (g/cm^3) can be set to any, the program will automatically calculate the database, then you need to delete (cut off) the rows for which ρ_s and θ exceed the boundary values from it $\rho_{s\ lim}$ and θ_{lim}.

In Table 3, the typical operating range of hydraulic devices is from 1.2 (feeding deslimers, hydrocyclones, etc.) to 1.6 g/cm^3 (marked in bold).

Additionally, the conditional viscosity index (100-1/v) is given, which is needed to determine the $\nu = f(\rho_s)$.

Also, the two columns on the right, reflect the flow rates of solid and liquid per unit (one device). As an example, the productivity of 50 tons/hour for one MD-5 deslimer in iron ore processing technology is taken. In the program, you can set any productivity, the amount of liquid to create a certain pulp density in the working area of the device is determined from formula (1) as:

$$Q_{liquid} = \frac{Q_{solid} \cdot (100 - \theta)}{\theta}$$

4. Conclusions

A mathematical apparatus for calculating hydraulic characteristics has been developed for the water suspension of fine mineral particles. Its peculiarity is that all characteristics are determined based on the measurement of only one parameter - the volume density of the suspension. This indicator is easily measured in practice and at processing plants serves to monitor the technological regime. An algorithm and a program have been developed to obtain a database of hydraulic characteristics θ, ε, β, ν in a wide range of suspension densities ρ_s, as well as to establish approximating dependences of characteristics on the density ρ_s. Based on the obtained hydraulic characteristics, calculations of the particle velocity in mineral suspensions are performed, which are the basis for the development of an optimal mode of hydraulic classification and separation and allow the design of various hydraulic apparatuses.

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МАТЕМАТИЧНА МОДЕЛЬ ВИЗНАЧЕННЯ ГІДРАВЛІЧНИХ ХАРАКТЕРИСТИК ТОНКОДИСПЕРСНИХ ВОДНИХ МИНЕРАЛЬНИХ СУСПЕНЗІЙ

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Анотація. Гідравлічні характеристики полідисперсних мінеральних суспензій такі як в'язкість, концентрація, порозність, об'ємний і ваговий вміст твердої та рідкої фази необхідні для розрахунку швидкості вільного та стиснутого осадження і спливання частинок різного складу та крупності. Зазначена швидкість є основою розрахунку гідравлічних апаратів - класифікаторів і сепараторів при збагаченні мінеральних пульп. Визначення гідравлічних характеристик вимагає безлічі експериментальних вимірювань з огляду на різний склад суспензій та режимів роботи апаратів. Відомі розрахункові формули є емпіричними та напівемпіричними. Теоретичні формули відомі лише для в'язкості, але вони обмежені концентрацією твердої фази 2–5 %. Метою роботи було розробити математичну модель визначення гідравлічних характеристик залежно лише від одного вимірюваного показника – об'ємної ваги проби або густини суспензії. Цей показник легко вимірюється на практиці, на збагачувальних фабриках він служить для моніторингу режиму експлуатації апаратів. У роботі використано пористу модель водної суспензії, що складається з дискретних частинок та класичні визначення гідравлічних характеристик. На підставі цього отримано визначальні формули, розроблено алгоритм та програму розрахунку характеристик. При використанні програми отримана база даних дозволяє встановити апроксимуючі залежності для вагового вмісту твердої фази θ, порозності ε, концентрації β, кінематичної в'язкості від щільності суспензії ρ_c в широкому діапазоні. Зазначені залежності дозволяють розраховувати гідравлічні характеристики для будь-якої зони апарату та різних режимів використовуючи лише один нескладний вимір густини пульпи ваговим методом. На підставі цього, наприклад, можна розрахувати швидкість осадження та спливання частинок та побудувати карту розподілу швидкостей та ефективності гравітаційного розділення частинок. Розроблена математична модель, алгоритм та програма розрахунку можуть використовуватись для оцінки оптимального режиму, контролю стабільності роботи відомого обладнання та проектування нових гідравлічних апаратів.

Ключові слова: мінеральна суспензія, щільність, в'язкість.